MATHEMATICS

4016/01

26 OCTOBER 2010

1 **(a)** $\frac{3}{35} \times 100\% = 8\frac{4}{7}\%$

(b) $17\frac{1}{2} \div 100 = \frac{7}{40}$

2 $\frac{3.93}{(7.47+3.02)\ 5.67} = 0.06607$

≈ 0.066 (2 sf)

3 $3x^3 - 12xy^2 = 3x(x^2 - 4y^2)$

$= 3x(x^2 - (2y)^2)$

$= 3x(x+2y)(x-2y)$

4 **(a)** $3^{23} \div 27 = 3^k$

$3^{23} \div 3^3 = 3^k$

$3^{23-3} = 3^k$

$23 - 3 = k$

$k = 20$

(b) $1 \div 2x^{-5} = 1 \div \frac{2}{x^5}$

$= 1 \times \frac{x^5}{2}$

$= \frac{x^5}{2}$

5 **(a)** $|t-(-3)| = |t+3|$

(b) Mean temperature $= \frac{-3+5+t}{3}$

$= \frac{2+t}{3}$

6 Let S denote the sum of money.

$\left(\frac{1}{3} - \frac{2}{2+3+4}\right)S = \left(\frac{1}{3} - \frac{2}{9}\right)S = \frac{S}{9} = 20$

$S = 9 \times 20 = 180$

7 **(a)** $2x^2 - 5x - 3 = (2x+1)(x-3)$

(b) $2x^2 - 5x - 3 = 0$

$(2x+1)(x-3) = 0$

$2x + 1 = 0$ OR $x - 3 = 0$

$2x = -1$ $\quad x = 3$

$x = -\frac{1}{2}$

8 $-2 \le 2x + 4 < 18$

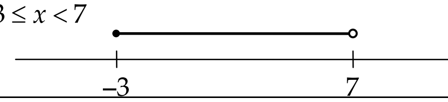
$-2 \le 2x + 4$ OR $2x + 4 < 18$

$-2 - 4 \le 2x$ $\quad 2x < 18 - 4$

$-6 \le 2x$ $\quad 2x < 14$

$-3 \le x$ $\quad x < 7$

$\therefore -3 \le x < 7$



9 **(a)** $A\hat{O}C = 180° - 2(28°)$

$= 124°$

(b) Let D be a point along the major arc AC.

$A\hat{D}C = 124° \div 2$ (angle at the circumference)

$= 62°$

$A\hat{B}C = 180° - 62°$ (cyclic quadrilateral)

$= 118°$

(c) $OAT = 90°$ (tangent to a circle)

$CAT = 90° - 28°$

$= 62°$

$ATC = 180° - 2(62°)$

$= 56°$

10 **(a)** $168 = 2 \times 2 \times 2 \times 3 \times 7$

$= 2^3 \times 3 \times 7$

(b) **(i)** LCM of 168 and 4900

$= 2^3 \times 3 \times 5^2 \times 7^2$

(ii) GCD of 168 and 4900

$= 2^2 \times 7$

$= 28$

11 **(a)** P(the total of the three numbers is 18)

$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$

(b) P(the three dice show the same number)

$= \frac{1}{216} \times 6$

$= \frac{1}{36}$

(c) Out of the three dice, one of them should be a 5.

There are $\binom{3}{1}$ possible ways to select that die.

P(the total of the three numbers is 17)

$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \binom{3}{1}$

$= \frac{1}{72}$

12 **(a)** $A' = \{-3, 3\}$

(b) $A \cap B = \{1, 2\}$

(c) $A \cup B = \{-2, -1, 0, 1, 2, 3\}$

13 **(a)** Since V is inversely proportional to P, $V = \frac{k}{P}$.

$$3 = \frac{k}{200}$$
$$k = 3 \times 200 = 600$$
$$\therefore V = \frac{600}{P}$$

When $P = 150$, $V = \frac{600}{150} = 4$.

(b) When $V = 5$, $P = \frac{600}{V} = \frac{600}{5} = 120$.

14 **(a)** $(3 \times 10^5) \times 10^3 = 3 \times 10^8$

(b) $\frac{1 \div (3 \times 10^8)}{10^{-9}} = 3\frac{1}{3}$

15 **(a)**
$$\begin{pmatrix} 7 & 2 & 3 \\ 6 & 6 & 2 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} (7)(3)+(2)(1)+(3)(0) \\ (6)(3)+(6)(1)+(2)(0) \end{pmatrix}$$
$$= \begin{pmatrix} 23 \\ 24 \end{pmatrix}$$

(b) The matrix represents the total points awarded for the two football teams.
The total points awarded for City and United is 23 and 24 respectively.

16 **(a)** $\frac{3a^2}{7bc} \div \frac{9a}{14b} = \frac{3a^2}{7bc} \times \frac{14b}{9a} = \frac{2a}{3c}$

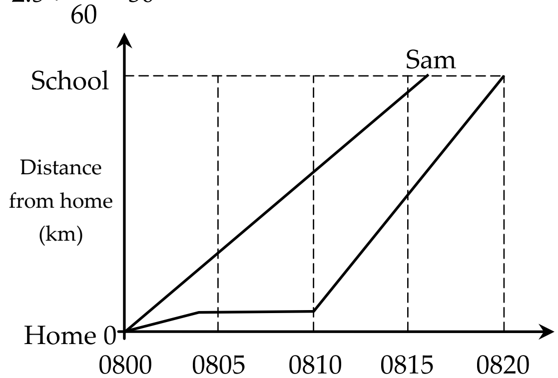
(b)
$$\frac{2x}{(2x-3)^2} - \frac{1}{2x-3} = \frac{2x}{(2x-3)^2} - \frac{2x-3}{(2x-3)^2}$$
$$= \frac{2x-(2x-3)}{(2x-3)^2}$$
$$= \frac{3}{(2x-3)^2}$$

17 **(a)** Time difference between 0804 and 0810
$= 6$ minutes

(b) $5.4 - 2.9 = 2.5$

(c) $2.5 \div \frac{5}{60} = 30$

(d)

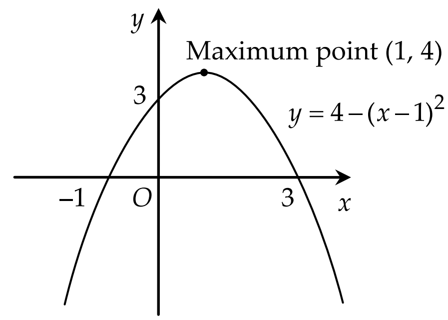


18 **(a)** 75

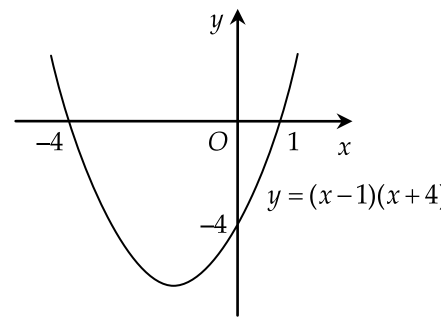
(b) 69

(c) The girls performed better because the mean marks of the girls are higher than that of the boys.

19 **(a)**



(b) **(i)**



(ii) $\frac{-4+1}{2} = -\frac{3}{2}$

Line of symmetry is $x = -\frac{3}{2}$

20 **(a)**
$$\text{Area of triangle } ABC = \frac{1}{2} \times (6-3) \times (5-1)$$
$$= 6$$

(b) Let x represent the x-coordinate of the point D.
$$\frac{AB+DC}{2} \times \text{height} = 14$$
$$\frac{(6-3)+(9-x)}{2} \times (5-1) = 14$$
$$\frac{12-x}{2} \times 4 = 14$$
$$12 - x = 7$$
$$x = 5$$

Since the y-coordinate of the point C is 5, the coordinates of the point D is $(5, 5)$.

(c) Let h represent the height of triangle ABE.
$$\frac{1}{2} \times AB \times h = 9$$
$$\frac{1}{2} \times (6-3) \times h = 9$$
$$h = \frac{9 \times 2}{3} = 6$$

$k = 1 + 6 = 7$ OR $k = 1 - 6 = -5$

21 **(a)** 1000 m = 1 km

$$\text{time taken} = \frac{\text{distance}}{\text{speed}} = \frac{1}{4} \text{ h}$$

$$= 15 \text{ minutes}$$

(b) $$\text{speed} = \frac{\text{distance}}{\text{time taken}} = \frac{1.3}{5 \div 60} = 15\frac{3}{5} \text{ km/h}$$

(c) $$\text{average speed} = \frac{\text{total distance}}{\text{total time taken}}$$

$$= \frac{1+1.3}{\frac{1}{4}+\frac{5}{60}}$$

$$= \frac{2.3}{\frac{1}{3}}$$

$$= 6.9 \text{ km/h}$$

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22 **(a)** $(2n-1)+2 = 2n+1$

$(2n+1)+2 = 2n+3$

The next two odd numbers are $2n+1$ and $2n+3$.

(b) **(i)** $(2n-1)+(2n+1)+(2n+3)$

$= 6n+3$

$= 3(2n+1)$

(ii) Since 3 is a factor of $3(2n+1)$, the sum is a multiple of 3.

(c) $(2n-1)^2+(2n+1)^2+(2n+3)^2$

$= (4n^2-4n+1)+(4n^2+4n+1)+(4n^2+12n+9)$

$= 12n^2+12n+11$

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23 **(a)** In Δs ALB and NLD,

$A\hat{L}B = N\hat{L}D$ (vertically opposite angles)

$A\hat{B}L = N\hat{D}L$ (alternate angles since $AB \parallel DN$)

$B\hat{A}L = D\hat{N}L$ (angle sum)

By the AAA property, Δs ALB and NLD are similar.

(b) ΔNDA

(c) ΔADB and ΔCBD

(d) **(i)** Consider triangles ALB and NLD.

Since $\frac{AB}{ND} = \frac{1}{3}$ and $AB = CD$,

$$\frac{AB}{CN} = \frac{AB}{ND-CD} = \frac{AB}{ND-AB} = \frac{1}{3-1} = \frac{1}{2}$$

(ii) $$\frac{\text{Area of } \Delta ABL}{\text{Area of } \Delta ADL} = \frac{1}{3}$$

(iii) $$\frac{\text{Area of } \Delta MLB}{\text{Area of } \Delta ALD} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

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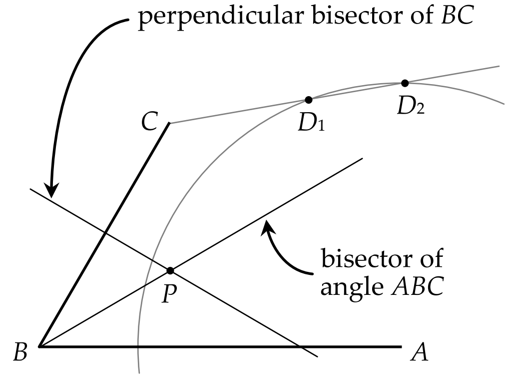
24 **(a)**



(b) See diagram

(c) The point P is equidistant from the lines AB and BC and equidistant from the points B and C.

(d) See diagram